UM-TH-00-08

Freiburg-THEP-007

A Note on Large Extra Dimensions.

R. Akhoury[(1)] **and J.J. van der Bij**[(1,2)]

[(1)] *The Randall Laboratory of Physics*

University of Michigan

Ann Arbor, MI 48109-1120

and

[(2)] *Fakultät für Physik*

Universität Freiburg

H. Herderstr. 3, 79104 Freiburg

Abstract

We study corrections to the photon-propagator in a recently proposed model, where gravity lives in 4+n dimensions and Standard Model fields in 4 dimensions. We find a correction to the formfactor of the photon that can be constrained by QED tests.

1 Introduction

Motivated in part by naturalness issues, recently, starting with ref. [1], there have been several attempts to develop models where the effects of gravity become large at energy scales of the order of a TeV - much lower than the traditionally accepted Planck mass. In fact, in a previous paper[2] by one of the authors it was suggested that the so-called hierarchy problem, the question why the Planck mass is much larger than the weak scale, could be solved by having gravity play a role at the electroweak scale. One of the implications of such a principle is the possibility of having large extra dimensions being present. This possibility has recently been presented in a simple form in ref.[3]. In this paper it was suggested that the standard model is confined on a D3 brane, but that gravity lives in the bulk of the (4+n) dimensional space-time, where the n dimensions are compactified, possibly with a large radius. From the formula $M_{Pl,4}^2 \approx R^n M_{Pl,4+n}^{n+2}$, the $4+n$ - dimensional Planck mass can be at the TeV scale if the size R of the higher dimensions is large enough. As prefigured in ref.[2] such scenarios lead to collider signals in the form of missing energy signals and anomalous couplings from higher dimensional operators. These effects could come from the production of higher dimensional gravitons and/or their tree-level exchange. Since such signals are absent in present-day colliders, limits can be put on the scale where the effects of higher-dimensional gravity become strong[4, 5]. Typically one finds limits of the order of a TeV for this scale. Astrophysical considerations allow one to limit the cases n=1 and 2.[6].

At first sight the model therefore appears to be in order, at least for $n > 2$. However, since it is non-renormalizable, one has to be careful that radiative effects do not destroy its consistency. Non-renormalizable models can often be used as effective Lagrangians, if the radiative corrections are sufficiently well behaved, so that the divergent higher-order effects can be absorbed in effective local operators, parametrizing our ignorance about the underlying fundamental dynamics. For example, this procedure works very well in pion physics, where the starting Lagrangian is the non-linear sigma-model[7, 8]. However the

divergences in higher-dimensional gravity are much more severe than in the non-linear sigma-model, so that explicit calculations are necessary to determine how far this model makes sense at the quantum level. Calculations involving loop-effects of higher-dimensional gravitons have been performed in [9, 10] for the LEP electroweak precision data, and in [11] for the g-2 factor of the muon. The authors in [9] and [10] both calculate corrections to the so called oblique parameters, coming from corrections to the vector-boson propagators. They reach different conclusions about the importance of the corrections, ref.[10] giving strong constraints, while ref.[9] gives much weaker ones. However both calculations ignore the non-oblique corrections, which is basically incorrect, since gravity couples to all particles, not only to vector-bosons. We therefore consider these results to be too uncertain to provide definite conclusions. In ref.[11] the g-2 factor of the muon was considered. Here it was found that the corrections are small, well within experimental bounds. However it is known from studies[12] on anomalous vector-boson couplings, that the g-2 factor of the muon is relatively insensitive to anomalous effects, that depend on the precise ultraviolet behaviour. The form-factor of the photon is a more sensitive quantity, therefore in the next section we calculate the radiative corrections to the photon-propagator due to higher-dimensional graviton exchange.

2 The Calculation

Such a calculation can be performed using the formalism of ref.[13]. The 4+n dimensional graviton-field Lagrangian is linearized and expanded in normal modes. The normal modes describe an infinity of 4-dimensional massive spin-2, spin-1 and spin-0 fields. The spin-1 fields decouple from ordinary matter, the spin-2 fields couple to the energy-momentum tensor $T_{\mu\nu}$ and the scalar fields to the trace of the energy momentum tensor. As we are interested in the coupling to photons, we can ignore the scalars completely, since the photon energy-momentum tensor is traceless. The propagator of the massive gravitons is derived in

[14]. Because we assume the extra dimensions to be large, the sum over the modes can be replaced by a density-integral. Assuming all extra dimensions to be circles with a radius R, the sum over graviton modes is replaced by the integral $\int dm^2 R^n m^{n-2}/((4\pi)^{n/2}\Gamma(n/2))$. At the one-loop level there are two types of graphs contributing. The tadpole graphs contain the 2-photon-2-graviton vertex, which is proportional to $k_\mu k_\nu - k^2\delta_{\mu\nu}$. Such diagrams, contain no further momentum dependence. Their contribution can therefore be absorbed in a wave-function renormalization of the photon. The remaining class of diagrams is non-trivial and contains the 2-photon-1-graviton vertices. It is strongly ultraviolet divergent and for low enough dimensions also infra-red divergent. As one also has to take care in the regularization to preserve the gauge-invariance of the photon, it is not very practical to evaluate this diagram with Feynman parameters. We therefore choose to evaluate the diagram by a dispersion relation. This way one preserves the symmetries that have to be present in the theory and at the same time keeps the physically relevant divergences. The form of the imaginary part is determined without ambiguities. One has however to choose an ultraviolet cut-off Λ for the spectrum, where one assumes new physics takes over. The infrared cut-off m_{gr} is taken as the mass of the lightest graviton.

With these assumptions the imaginary part of the photon two-point function becomes ($k^2 = \; -s$):

$$Im\Pi_{\mu\nu}(s) = (\delta_{\mu\nu}k^2 - k_\mu k_\nu)(-\frac{1}{9M_{Pl,4+n}^{n+2}}) \times (\frac{480 s^{(n+2)/2}}{(n-4)(n+2)(n+4)(n+6)} - \frac{s^3}{n-4}m_{gr}^{n-4} + \frac{10}{n+2}m_{gr}^{n+2} - \frac{15}{n+4}\frac{m_{gr}^{n+4}}{s} + \frac{6}{n+6}s^{-2}m_{gr}^{n+6}) \tag{1}$$

As we assume R to be large the last three terms in the above expression can be ignored. This expression is gauge invariant. In the following it will be convenient to introduce as usual the scalar $\Pi(k^2)$ by

$$\Pi_{\mu\nu}(k) \;=\; (\delta_{\mu\nu}k^2 - k_\mu k_\nu)\Pi(k^2) \tag{2}$$

It is for this object which is free of kinematic singularities that a dispersion relation can be written. Indeed, the real part of $\Pi(k^2)$ can now be determined by the relation:

$$Re\Pi(s) = \frac{1}{\pi} \times \int_{m_g^2}^{\Lambda^2} \frac{ds'}{s'-s} Im\Pi(s') \tag{3}$$

The full photon propagator ($\Delta'_{\mu\nu}$), in a general gauge is then given by

$$\Delta'_{\mu\nu} \;=\; \frac{\delta_{\mu\nu}}{k^2 - k^2\Pi(k^2)} - \frac{k_\mu k_\nu}{k^2} \frac{(1-\alpha)\Pi(k^2)+\alpha}{k^2 - k^2\Pi(k^2)} \tag{4}$$

Note that in the Landau gauge ($\alpha = 1$) the above has a particularly simple form:

$$\Delta'_{\mu\nu}(k) \;=\; \frac{(\delta_{\mu\nu} - k_\mu k_\nu / k^2)}{k^2 - k^2\Pi(k^2)} \tag{5}$$

Since the polarization tensor is transverse, no subtractions are needed for the photon mass which stays zero. This is the reason for choosing the scalar function $\Pi(k^2)$ as above with its appropriate kinematic factor. However, because of the high powers of s in imaginary $\Pi(k^2)$ a large number of subtractions would be needed, each corresponding to a higher dimensional operator, coming from a taylor series expansion of $Re\Pi(k^2)$ in powers of k^2. The coefficients of these terms are then not fixed. As we wish to consider the cut-off to be a physical quantity, as in ref.[8-10], we want to use the integral to determine these coefficients. The lowest order, term ($\Pi(0)$) would correspond to a wave-function renormalization. So the first non-trivial term is the k^4 term in the (inverse) photon two-point function. It is given by $\Pi'(s=0)$. To be more precise the coefficient β in the contribution to the polarization tensor $\beta k^2(\delta_{\mu\nu}k^2 - k_\mu k_\nu)$ is given by:

$$\begin{aligned}
n > 4 \qquad \beta &= \left(\frac{\Lambda^n}{9\pi M_{Pl,4+n}^{n+2}}\right) \frac{960}{n(n-4)(n+2)(n+4)(n+6)} \\
n = 4 \qquad \beta &= \left(\frac{\Lambda^4}{18\pi M_{Pl,4+n}^{n+2}}\right)\left(\ln\left(\frac{\Lambda^2}{m_g^2}\right) - 1/2\right) \\
n < 4 \qquad \beta &= \left(\frac{1}{9\pi M_{Pl,4+n}^{n+2}}\right) \frac{m_{gr}^{n-4}}{2(4-n)} \Lambda^4
\end{aligned} \tag{6}$$

3 Discussion.

The results in the above section clarify some of the questions raised in the literature. The question has been raised in ref.[8], whether radiative contributions can actually grow with the cut-off, or whether they are always suppressed by $1/M^2_{Pl,4+n}$. We see for large n a strong cut-off dependence. It is only when we put $\Lambda = M_{Pl,4+n}$, that the $1/M^2_{Pl,4+n}$ behaviour arises. In principle the cut-off Λ is the string scale, where higher spin resonances start appearing. For the field theory calculation of this paper to be sensible this string scale should be higher than the Planck scale, as otherwise operators coming from the higher-spin fields would be more important than the graviton contribution. In ref. ([5]) it has been argued that actually the string scale is lower than the Planck scale by a factor 1.6-3. If this is indeed the case, one cannot use the radiative corrections reliably to give limits on the Planck mass. For comparison with experiment, we will however take $\Lambda = M_{Pl,4+n}$.

The second question is the meaning of the infrared divergences, in particular the power divergences in m_{gr}. In ref.([10]) the power divergences were present, however it was argued that they should be absent in the observables because gravity is IR finite. In ref.([9]) they were regularized, with the argument that non-perturbative effects would make them disappear. It is not clear to us what these effects are. In particular, it is disturbing that the imaginary part of the photon 2 point function is not finite in the limit $m_{gr} \to 0$ for $n < 4$. We feel that this singularity is not really an infrared one in the sense that it does not necessarily arise due to long wavelength excitations. The singular behaviour in Eq. (1), for example, comes from the $k_\mu k_\nu k_\lambda k_\rho / m^4_{gr}$ term in the graviton propagator. If all invariances in the theory were kept intact then such terms do not contribute to singular behaviour in physical observables, -for example, if current conservation is preserved as in massive photon QED then a term like $k_\mu k_\nu / m^2_{ph}$ in the photon propagator does not give singularities in observables. In the brane scenario, the brane position breaks the higher dimensional general coordinate invariance. Thus even though conventional Kaluza Klein theories or string theo-

ries are consistent in terms of preserving invariances, it is not clear if the brane scenarios in their current form are. We believe that the singular behaviour in Eq. (1) is a reflection of this. That this depends on the number of extra dimensions could then be simply a reflection of the fact that the bigger the number of extra dimensions, the harder it is for higher dimensional gravitons to find the D3 brane. Comparing with ref.([10]), we notice that in the photon structure the power divergences are similar to those in the previously calculated S,T,U parameters for $n = 3$. It is not so clear to us that terms like the second one in Eq. (1) can be completely ignored even after a proper understanding of the theoretical issues involved.

A limit can be given by comparing the QED-test on the renormalized photon-propagator $\frac{\delta_{\mu\nu}}{k^2 - k^4/\Lambda_{QED}^2}$. One has as a limit $\Lambda_{QED} \approx 200GeV$ from colliders. Putting $\beta = 1/\Lambda_{QED}^2$ gives a limit on the Planck mass. For $n > 3$ this limit is not competitive with the limits given for the S,T,U parameters in refs.([10], [9]) or with the direct invisible energy search. Thus due to the large denominator in Eq.(6) we get a limit at best $M_{Pl,4+n} > 30GeV$. For the case of $n \leq 3$, somewhat more useful limits can be set. If we naively keep the terms which are singular as $m_{gr} \to 0$ for $n < 4$ then one gets large values, greater than a TeV, for the Planck mass.

In conclusion, at the moment, theories with large extra dimensions appear to be consistent at least for large enough n. The problem of singularities in physical observables as $m_{gr} \to 0$ needs to be studied at a theoretical level.

4 Acknowledgements

This work was supported by the NATO grant CRG 970113, by a University of Michigan Rackham grant to promote international partnerships and by the US Department of Energy. We would like to thank M. Einhorn, E. Yao and O. Yakovlev for discussions.

References

[1] I. Antoniadis, Phys. Lett. B246, 317 (1990).

[2] J.J. van der Bij, Int. J. Phys. 1, 63 (1995), hep-ph/9507389.

[3] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. lett. B429, 263 (1998).

[4] S. Nussinov and R. E. Schrock, Phys. Rev. D59, 105002 (1999); E. A. Mirabelli, M. Perelstein and M. Peskin, Phys. Rev. Lett., 82, 2236 (1999); G. Shiu, R. Schrock and S. H. H. Tye, Phys. Lett, B458, 274 (1999); G.F. Giudice, R. Rattazzi and J.D. Wells, Nucl. phys. B544, 3 (1999).

[5] S. Cullen, M. Perelstein, and M. Peskin, SLAC-PUB-8319, SU-ITP-99/53, hep-ph/0001166.

[6] S. Cullen and M. Perelstein, Phys. Rev. Lett. 83, 268 (1999); L. Hall and D. Smith, Phys. Rev. D60, 085008 (1999); V. Barger, T. Han, C. Kao and R. J. Zhang, Phys. Lett. B461, 34 (1999).

[7] S. Weinberg, Physica A96, 327 (1979).

[8] J. Gasser and H. Leutwyler, Ann. Phys. 158, 142 (1984); Nucl. Phys. B250, 465 (1985).

[9] T. Han, D. Marfatia and R. J. Zhang, MADPH-00-1156, hep-ph/0001320.

[10] P. Das and S. Raychaudhuri, IITK-HEP-99-53, hep-ph/9908205.

[11] M.L. Graesser, LBNL-42812, UCB-PTH-99/03, hep-ph/9902310.

[12] J.J. van der Bij, Phys. Rev. D35, 1088 (1987).

[13] T. Han, J.D. Lykken and R.-J. Zhang, Phys. Rev. D59, 105006 (1999).

[14] M.J.G. Veltman, in Methods in Field Theory, Les Houches 1975, North-Holland.